

Chapter 1 — What Proportional Reasoning Means — Teacher Guide

Chapter at a glance

Lesson duration	4 lessons, 40–60 minutes each (see pacing table below); can compress to 3 for a strong class
Materials	12 counters/coins/tiles per student, a strip of paper or string, a ruler, a coordinate grid, calculator (after estimation only)
Launch question	“Two shops sell notebooks. One increases steadily by \$2. Is it automatically proportional?”
Key representation	The double number line and table pair — keep one visible throughout instruction
Anticipated responses	Correct: names the constant quotient and the zero case. Partial: notices the pattern but not the quotient. Incorrect: “proportional because it goes up by /4 each time.”
If the readiness check fails	Reteach multiplication/division fact fluency and reading a table before starting Part A; do not skip the concrete build
Talk prompt	“What stays the same: the difference or the quotient?”
Exit ticket	3 items — constant of proportionality, reject a non-proportional equation, unit-rate word problem (see Section 14)
Reteach / extend	Reteach: unit strip + zero case, whole-number rates only. Extend: fractional constants, compare two solution methods.

Chapter purpose

This chapter establishes the conceptual foundation for the entire course. Students should leave understanding that proportionality is a constant multiplicative relationship, not a synonym for “the numbers go up” and not a synonym for cross-multiplication.

The chapter deliberately connects five representations: words, unit strips, double number lines, tables, equations, and graphs. Students should be expected to move between representations and explain what the constant means in context.

Suggested pacing

Lesson	Focus	Suggested time
Lesson 1	Launch, additive versus multiplicative change,	45–55 minutes

Lesson	Focus	Suggested time
	unit rate, and proportional definition	
Lesson 2	Tables, double number lines, equations, graphs, and worked examples	45–55 minutes
Lesson 3	Guided practice, misconception diagnosis, and Worksheet A	40–50 minutes
Lesson 4	Worksheets B and C, Tripwire Check, Exit Ticket, and reteach/extension	45–60 minutes

The chapter can be compressed for a strong class, but the visual representation and discussion stages should not be removed for intervention students.

Materials

Use the chapter diagrams, a ruler or measuring strip, sticky notes or counters, a coordinate grid, and two sets of cards labeled with input/output pairs. If available, provide a calculator only after students have estimated and identified the relationship.

Teacher launch

Display the two shop tables without naming either relationship proportional or non-proportional. Ask students to make a prediction silently, then compare their reasoning with a partner. Listen for the following responses:

- “Both are proportional because both increase by the same amount.”
- “Shop B is not proportional because it starts at \$5.”
- “Shop A has a constant price per notebook.”
- “The output difference is not enough information.”

Do not correct every response immediately. Record the claims and return to them after students have used the unit strip, table, and zero case.

Key teacher questions

Purpose	Prompt
Identify quantities	“What does each number represent? What are the units?”
Surface multiplicative reasoning	“How many times as much is the output?”
Distinguish additive reasoning	“What stays the same: the difference or the quotient?”
Interpret the constant	“What does k mean in this situation?”
Use the zero case	“What should happen when the input is zero?”
Connect representations	“Where do you see the same unit rate in

Purpose	Prompt
	the table, equation, and graph?"
Encourage checking	"Is your answer reasonable compared with a nearby known value?"
Promote method choice	"Which method makes the relationship easiest to see?"

Anticipated errors and responses

Student response	Interpretation	Immediate response
"It is proportional because it adds 2."	Additive thinking is replacing multiplicative thinking.	Ask for $y \div x$ at two nonzero inputs and compare the quotients.
"The starting fee is just part of the rate."	Fixed amount and variable rate are being combined.	Ask for the cost at zero input and write $y = 2x + 5$.
"A straight line is proportional."	Graph shape is being used without checking the origin.	Compare $y = 3x$ and $y = 3x + 5$; focus on (0,0).
"The unit rate is 12 because 4 notebooks cost \$12."	Total amount is being mistaken for rate.	Ask, "How much is one notebook?" and preserve the units.
"Cross-multiplication is always the first step."	Procedure is replacing relationship identification.	Ask the student to solve with a unit rate or table and explain the meaning first.

Differentiation

Support for students who are not yet secure

Begin with unit strips and physical grouping before using equations. Keep the input and output quantities in separate labeled rows. Use only whole-number rates at first, then move to decimals. Ask the student to complete one row at a time and say the units aloud: "\$3 for 1 notebook," "\$6 for 2 notebooks." Return to the zero case often.

For students with language needs, provide sentence frames:

- "The input represents _____."
- "The output represents _____."
- "The unit rate is _____ per _____."
- "The relationship is proportional because _____."
- "The relationship is not proportional because _____."

Extension for students ready to generalize

Ask students to compare two methods for the same problem, create a proportional and non-proportional pair with the same additive pattern, interpret fractional constants of proportionality, and explain why (0,0) and (1,k) are especially informative points on a proportional graph.

Mastery criteria

A student has mastered Chapter 1 when the student can classify a relationship and justify the classification, using a constant multiplier, a unit rate, an equation, a graph, or a zero-case argument. The student can solve for a missing value while keeping the units straight. The student can also explain why a plausible alternative method or answer is wrong.

A correct numerical answer alone is not sufficient evidence of mastery when the task is designed to assess reasoning.

Revision Support: Expected Ideas and Teacher Moves

Use this compact guide with the new Cumulative Connection and Strategy Studio tasks. Accept equivalent representations when the relationship, units, and reasoning are correct.

Cumulative Connection: A table has x-values 0, 1, 2, 4 and y-values 0, 3, 6, 12. State the multiplier. Decide whether 2, 4, 6, 8 is additive, multiplicative, both, or neither when the inputs are 1, 2, 3, 4. Plot or describe the point that must be included in every proportional graph. Explain why a constant difference alone does not prove proportionality.

Strategy Studio guidance: Expected ideas: multiplier 3; sequence can be proportional when paired with $x = 1, 2, 3, 4$, but students must name the paired inputs; the origin (0,0) is required for a proportional graph; B fails because 11 is not 3 times 4. Treat the “same first values, different later value” task as an opportunity to discuss what evidence is sufficient.

Transfer Bridge Teacher Guidance

Use the four tasks to distinguish imitation from transfer. Ask students to name the relationship, keep units visible, compare representations, and explain or repair an error. Accept equivalent methods when the structure and interpretation are correct.

Expected guidance: Familiar: gives 0, 4, 8, 20 and multiplier 4. Altered: the second table has constant difference 6 but begins at $y = 1$, so it is not directly proportional. Non-routine: Shop A is $y = 6x$; Shop B is . Repair: direct proportionality requires a constant multiplier and the zero case $y = 0$ when $x = 0$.

Record whether the difficulty is classification, representation, operation, computation, units, interpretation, or checking. Do not assign more routine practice until the source of the error is identified.

Discussion and evidence note

Ask students to name the relationship before calculating, compare at least two representations when appropriate, and revise an explanation after hearing another strategy. Record whether the difficulty is classification, representation, operation, computation, units, or checking.

Answer Key

Launch: Two shops

1. Shop A.
2. Shop A has a constant cost of \$3 per notebook; Shop B has a constant increase of \$2 but also a \$5 starting fee.
3. No. Shop A is proportional; Shop B is not because it starts at \$5 and has equation $C = 2n + 5$.
4. **Both** shops charge \$15 for 5 notebooks (Shop A: $3 \times 5 = 15$; Shop B: $2 \times 5 + 5 = 15$). This is a coincidence at this one value, not evidence that the shops work the same way.
5. **Shop B.** At 10 notebooks, Shop A costs \$30 and Shop B costs \$25, so Shop B is the one that charges \$25. Comparing a second value is exactly how the coincidence in Question 4 gets caught.

Additive versus multiplicative change

1. Additive and proportional if interpreted as $2x$ for inputs 1, 2, 3, 4; the sequence itself needs its input values to determine proportionality.
2. Multiplicative, not additive.
3. Additive and proportional if the inputs are 1, 2, 3, 4 and outputs are $5x$.
4. Additive, not proportional for the usual inputs 1, 2, 3, 4.
5. Neither a constant difference nor a constant multiplier.
6. Additive and proportional: each term is times the term number, so $y \div x$ is always .

Watch for: students who label every steady-looking sequence “multiplicative” without actually checking , or who assume a sequence “can’t be proportional” because it contains fractions. If either

pattern shows up, return to Question 6 and compute for each pair before moving on.

Worksheet A

1. **\$0, \$4, \$8, \$20, \$32.** Likely wrong answer: \$0, \$4, \$8, \$12, \$16 — fills the blanks by adding \$4 to the previous blank instead of computing $n \times \$4$ for each column, so the jump from 2 items to 5 items gets missed. Misconception: treating the table as a running total instead of an independent multiplication at each n . Next move: cover the rest of the table and ask “what is $5 \times \$4$?” on its own.
2. **\$4 per item.** Likely wrong answer: “4” with no units, or “0.25” (the rate inverted). Misconception: dropping units or inverting the rate’s direction. Next move: ask “4 what, per what?” and check against a known pair from the table.
3. **$C = 4n$.** Likely wrong answer: . Misconception: writing the additive form instead of the multiplicative form. Next move: test the equation at $n = 1$ does it give \$4?
4. **One item costs \$4.** Likely wrong answer: restates the coordinates (“it’s at 1 and 4”) without saying what they mean. Misconception: treating the point as an abstract pair instead of a rate statement. Next move: ask what each axis represents before asking about the point.
5. **Zero items cost \$0.** Likely wrong answer: “the graph starts at zero,” with no link to the real-world meaning. Misconception: not connecting the origin to the zero-item case. Next move: ask “what does buying zero items mean for the cost?”
6. **Proportional; for every nonzero input.** Likely wrong answer:

- “proportional,” with no supporting calculation shown. Misconception: judging proportionality by the look of the pattern instead of computing the quotient. Next move: require the quotient shown for at least two pairs before accepting the verdict.*
7. **Not proportional; the difference is +2, but the quotients are not constant.** Likely wrong answer: *“proportional, because it goes up by 2 each time.”* Misconception: *additive change mistaken for multiplicative change, the chapter’s core misconception. Next move: compute $y \div x$ at two nonzero inputs and compare.*
 8. **Not proportional; at 0 kilometres the cost is \$4, not \$0.** Likely wrong answer: *“proportional, because it’s \$2 per kilometre.”* Misconception: *ignoring the fixed starting fee. Next move: ask what the cost is at 0 kilometres.*
 9. **\$3 per notebook.** Likely wrong answer: *\$75 (multiplies 15×5 instead of dividing) or about \$0.33 (inverts the division).* Misconception: *choosing the wrong operation or direction. Next move: ask whether one notebook should cost more or less than \$15.*
 10. **\$24.** Likely wrong answer: *\$40 (multiplies 8×5 , reusing the given notebook count as if it were the rate).* Misconception: *using a number given in the problem as the rate instead of the computed unit rate. Next move: ask “what is the cost of ONE notebook first?”*
 11. **60 km/h.** Likely wrong answer: *“60” with no units, or a value from computing $3 \div 180$.* Misconception: *dropping units or inverting the rate. Next move: check “60 what per what?”*
 12. **300 km.** Likely wrong answer: *65 (adds $60 + 5$ instead of multiplying).* Misconception: *combining the rate and the time additively. Next move: “if 1 hour is 60 km, what is 5 hours?”*
 13. **Answers vary; the table must include a zero row and the same quotient $y \div x$ at every nonzero pair.** Common flaw: *the table looks proportional but the multiplier is not actually constant across all rows, or the zero row is missing. Next move: have the student check $y \div x$ for every pair, not just the first.*
 14. **An additive pattern keeps a constant difference; a proportional relationship keeps a constant multiplier (unit rate).** Common flaw: *describes only one of the two ideas, or uses “goes up steadily” to describe both. Next move: ask for one numeric example of each kind.*

Worksheet B

1. **Proportional; , and the quotient is 7.** Likely wrong answer: *reaches “proportional” correctly, but only by checking that the difference is constant (+7), not the quotient.* Misconception: *this particular table’s numbers make additive and multiplicative reasoning agree, which can quietly reinforce the additive shortcut. Next move: ask “would this still be proportional if the first value were 2 instead of 0? Test that idea on Question 2.”*
2. **Not proportional; starts at 2, equation .** Likely wrong answer: *“proportional, because it goes up by 3 each time.”* Misconception: *additive change mistaken for multiplicative change. Next move: compare $y \div x$ at $x = 1$ and $x = 2$.*

3. **Proportional; $y \div x = 12$ for every pair.** Likely wrong answer: “not proportional, because the inputs (1, 2, 4, 6) aren’t evenly spaced.” Misconception: believing proportional tables need consecutive whole-number inputs. Next move: compute $y \div x$ for each pair regardless of spacing.
4. **Not proportional; the difference is 3, but the quotient changes.** Likely wrong answer: “proportional, because it adds 3 each time.” Misconception: additive-for-multiplicative, again. Next move: $y \div x$ at $x = 1$ versus $x = 2$.
5. **Proportional; it has the form $y = 2.5x$.** Likely wrong answer: “not proportional, because 2.5 isn’t a whole number.” Misconception: believing the constant of proportionality must be a whole number. Next move: test a value, at $x = 2$, does $y = 5$, and does $y \div x = 2.5$ hold?
6. **Not proportional; the added 4 is a starting amount.** Likely wrong answer: “proportional, because it still has $2.5x$ in it.” Misconception: focusing on the multiplicative term and overlooking the added constant. Next move: evaluate at $x = 0$, does $y = 0$?
7. **\$12 per hour.** Likely wrong answer: about \$0.08 (inverts the division) or \$77 (subtracts instead of divides). Misconception: choosing the wrong operation. Next move: “is the hourly rate more or less than \$84?”
8. **\$144.** Likely wrong answer: \$96 or another sum from combining 84 and 12 additively instead of multiplying rate $\times$ hours. Misconception: combining given numbers additively instead of scaling. Next move: “hours $\times$ rate — what does that give you?”
9. **10 cups.** Likely wrong answer: 19 cups (adds the difference between 10 and 25 people, which is 15, directly onto the 4 cups). Misconception: applying an additive jump to a multiplicative scaling situation. Next move: “does adding 15 to the cups match how 10 relates to 25 people? Try the unit rate instead.”
10. **\$22.** Likely wrong answer: \$66 (multiplies 11 bottles $\times$ \$6 instead of finding the unit rate first). Misconception: skipping the unit-rate step and multiplying the total price directly by the new count. Next move: “how much is ONE bottle?”
11. **A proportional relationship would cost \$0 at 0 kilometres; the \$5 starting fee makes the equation $C = 2d + 5$, so the relationship is not proportional even though it increases by \$2 per kilometre.** Common flaw: says “because of the starting fee” without connecting it to the zero-case test. Next move: require the answer to state the cost at $d = 0$.
12. **The first part changes from 3 to 6 by a factor of 2 (not by adding 3), so the second part must also be multiplied by 2: $5 \times 2 = 10$. The correct equivalent ratio is 6:10, not 6:8.** Common flaw when correcting: finds a scale factor from the wrong pair of numbers (for example, dividing 8 by 5) and produces a second, still-inconsistent ratio. Next move: label which number is “first” and “second” in both ratios before finding the factor.

13. **Answers vary; must show a context, table, and equation of the form with a consistent quotient at every pair.** *Common flaw: the decimal rate gets rounded differently row to row, so the table looks proportional but isn't exactly. Next move: check $y \div x$ at every row, not just one.*
14. **Answers vary; must show a context with a starting amount that is not zero and a constant difference of 3, with an explanation that references the zero case or the changing quotient.** *Common flaw: the student's own example is accidentally proportional (starts at 0). Next move: ask what happens in their example when the input is 0.*

Worksheet C

1. **\$2.40 per folder; equation $C = 2.4f$; 15 folders cost \$36.** *Likely wrong answer: an arithmetic slip in 2.4×15 (for example, \$34 or \$38). Misconception: a decimal-placement slip rather than a conceptual error. Next move: estimate first ($15 \times \$2 \approx \30 , so the exact answer should be a bit more), then check by adding $10 \times \$2.40$ and $5 \times \$2.40$.*
2. **6 cups of concentrate.** *Likely wrong answer: applies the scale factor to the wrong quantity, or reverses the ratio direction. Misconception: confusing which quantity is being scaled from and which is being scaled to. Next move: "from 7 to 21 cups of water, what's the scale factor? Apply that same factor to the concentrate."*
3. **\$5, \$7, \$9, \$11; not proportional, since the cost at 0 kilometres is \$5, not \$0.** *Likely wrong answer:*

fills the table correctly but says "proportional, because it's \$2 per kilometre." Misconception: the recurring fixed-fee blind spot. Next move: look specifically at the (0, __) entry.

4. **No. The output has a constant difference of 4, but the quotient changes, and the relationship does not pass through (0,0).** *Likely wrong answer: "yes, proportional, because it adds 4 each time." Misconception: additive-for-multiplicative, worth re-testing this late in the chapter to confirm it's resolved. Next move: $y \div x$ at two points.*
5. **Store A: \$3.50/kg. Store B: about \$3.17/kg. Store B has the lower unit price.** *Likely wrong answer: "Store A is cheaper, because \$14 is less than \$19." Misconception: comparing total prices instead of unit prices when the quantities being bought aren't equal. Next move: "are you comparing the same amount of rice in both stores?"*
6. **36 km; proportional because each 1 cm represents the same 8 km, including at 0.** *Likely wrong answer: an arithmetic slip on 4.5×8 (for example, 32 or 40) from mishandling the decimal. Misconception: treating 4.5 as though it were 4, or misplacing the decimal in the product. Next move: break it into $4 \times 8 = 32$ plus $0.5 \times 8 = 4$, total 36.*
7. **No. The line begins at (0,5), so the 5 is likely a fixed starting amount that exists even when the input is 0.** *Likely wrong answer: "yes, proportional, because it's a straight line." Misconception: line shape mistaken for proportionality, without checking the origin, the chapter's own named*

- misconception. Next move: "does the line pass through (0,0)?"*
8. **Missing value is 6 (since $y = 3x$); proportional, with constant of proportionality 3.** *Likely wrong answer: estimates the missing value by interpolating between the given 3 and 15 (for example, guessing 9) instead of applying the rule. Misconception: surface-level "in-between" guessing instead of using $y = 3x$ directly. Next move: "what rule connects x and y ? Apply it to $x = 2$."*
 9. **Student A is correct; nine pens cost $9 \times \$2.50 = \22.50 .** *Likely wrong answer: sides with Student B, because " $4:10 = 9:15$ " looks like it follows the same pattern. Misconception: doesn't recognize that Student B scaled by adding instead of multiplying, the same error as Question 12 in Worksheet B. Next move: check whether 9 pens really relates to 4 pens the way 15 relates to 10; test with the unit rate instead.*
 10. **Answers vary; must describe where a fixed fee is added and explain why that breaks the zero-case or constant-quotient test.** *Common flaw: describes a fee appearing but doesn't explain why it breaks proportionality. Next move: ask the student to state the cost at the input value where the model changes.*
 11. **Answers vary; must show at least three representations (for example, table, equation, graph) for $y = 4x$ and a sentence connecting them through the same constant, 4.** *Common flaw: shows the representations but skips the connecting sentence. Next move: ask "where do you see the 4 in each one?"*
 12. **Answers vary; the non-proportional table must fail the zero-case or constant-quotient test even though both tables increase.** *Common flaw: the "non-proportional" table is accidentally proportional (starts at 0 with a constant ratio). Next move: check the student's own non-proportional table against both tests before accepting it.*
 13. **Unit rate is 2 miles per hour (); in 1 hour, the runner travels 2 miles; proportional, since the pace is constant and 0 hours gives 0 miles.** *Likely wrong answer: adds the fractions instead of dividing them, or answers by multiplying instead of dividing. Misconception: treating a fractional rate the same way as multiplying two fractions instead of dividing them. Next move: ask "how many quarter-hours fit inside a whole hour? What does the runner do in each one?"*
 14. **\$18. Mia's age and the fact that she paid with a \$20 bill are not needed to find the total cost.** *Likely wrong answer: works the \$20 into the calculation somehow (for example, finds change) even though the question only asks for the total cost. Misconception: assuming every number in a word problem must be used. Next move: ask "what does the question actually ask for? Which numbers answer that?"*
 15. **Answers vary; must describe a situation where 4.5 is a rate ("dollars per item," "hours per item," or similar) and n counts the items, with the meaning of 4.5 stated using units.** *Common flaw: gives a situation but never states what 4.5 means, or states it without units. Next move: ask the*

student to finish the sentence “4.5 means ___ per ___.”

Extend your thinking

1. $y = 22$; 2.75 means 2.75 units of output for every 1 unit of input.
2. Yes; for example, $C = 1.5n$ means \$1.50 per item.
3. No. The two points suggest a constant rate of 5, but a third point, the zero case, or the context is needed to confirm proportionality.
4. A taxi with a \$5 starting fee and a constant cost per kilometre.
5. A proportional line has the form and passes through (0,0); a non-proportional straight line can have a fixed starting amount, such as with $b \neq 0$.

Tripwire Check

Part 1: No. The output has a constant difference of 2, but the quotient changes; also, (0,6) is not (0,0). Part 2: No. The \$4 starting fee creates a fixed amount, so the equation is not .

Exit Ticket

1. $k = 6$; it means 6 output units for every 1 input unit.
2. No; has a starting amount of 3 and does not pass through the origin.
3. \$0.90 per apple; \$7.20 for 8 apples.

choice, and a repeated “understand–represent–solve–check–explain” routine. The structure reflects evidence-informed guidance that emphasizes visual representations, discussion, reflection, and problem-solving processes in middle-grade mathematics. The chapter also supports Grade 7 proportional-relationships expectations involving unit rates, tables, graphs, equations, constant of proportionality, interpretation of (0,0) and (1,k), and multistep applications [3].

References

[1]: <https://ies.ed.gov/ncee/wwc/PracticeGuide/16> “Improving Mathematical Problem Solving in Grades 4 Through 8,” Institute of Education Sciences, What Works Clearinghouse.

[2]: <https://nel.moe.edu.sg/la/numeracy/how-can-you-do-it/using-concrete-pictorial-abstract-cpa-approach/> “Using Concrete–Pictorial–Abstract (CPA) Approach,” Singapore Ministry of Education.

[3]: <https://www.thecorestandards.org/Math/Content/7/RP/> “Grade 7: Ratios & Proportional Relationships,” Common Core State Standards Initiative.

Teacher design note

This chapter is intentionally built around multiple representations, explicit misconception analysis, visible method

